Equivariant Chern characters and applications to K- and L-theory

by

Wolfgang Lück (Münster)

in Edinburgh

Tuesday, November 9-th

We construct for G-equivariant K-homology an equivariant Chern character, which is an isomorphism and lives over the ring $\mathbb{Z} \subset \Lambda^G \subset \mathbb{Q}$ obtained from the integers by inverting the orders of all finite subgroups of G. Its source is essentially given by group homology of the centralizers and representation rings of finite cyclic subgroups of G. We show that the Baum-Connes Conjecture implies a modification of the Trace Conjecture due to Baum and Connes, which says that the image of the standard trace $K_0(C_r^*(G)) \to \mathbb{R}$ takes values in Λ^G.

The Chern character mentioned above is a special case of an equivariant Chern character which can be applied to any equivariant (co-)homology theory, for instance also to those appearing as sources of the assembly maps in the Farrell-Jones Isomorphism Conjecture whose target are K-and L-groups of group rings. They also enter in the extension of the proof of the K-theoretic Novikov Conjecture due to Bökstedt-Hsiang-Madsen from the family of the trivial subgroup to the family of finite subgroups thus detecting a much larger portion in the algebraic K-theory of an integral group ring. They also lead to a computation of $K^*(BG) \otimes_{\mathbb{Z}} \mathbb{Q}$ for infinite groups under certain finiteness conditions.

Survey on classifying spaces for families of subgroups.
by
Wolfgang Lück (Münster)
in Southampton
Wednesday, November 10-th

We define for a topological group G and a family of subgroups F two versions for the classifying space for the family $\mathcal{F}$, the G-CW-version $E_{\mathcal{F}}(G)$ and the numerable G-space version $J_{\mathcal{F}}(G)$. They agree if G is discrete, or if G is a Lie group and each element in $\mathcal{F}$ compact, or if $\mathcal{F}$ is the family of compact subgroups or of compact open subgroups. We discuss special geometric models for these spaces for the family of compact open groups in special cases such as almost connected groups G and word hyperbolic groups G. We deal with the question whether there are finite models, models of finite type, finite dimensional models. We also discuss the relevance of these spaces for the Baum-Connes Conjecture about the topological K-theory of the reduced group C^*-algebra, for the Farrell-Jones Conjecture about the algebraic K- and L-theory of group rings, for Completion Theorems and for classifying spaces for equivariant vector bundles and for other situations.

Survey on L^2-invariants
by
Wolfgang Lück (Münster)
in London
Friday, November 12-th

We will introduce the basic L^2-invariants, namely the L^2-Betti numbers and will discuss their basic properties. Their definition is based on some functional analysis including finite von Neumann algebras and Hilbert modules over them.

We will discuss applications to questions in topology, geometry and group theory such as S^1-actions on aspherical closed manifolds, the Hopf Conjecture for non-positive curved closed Riemannian manifolds, deficiency of groups, fundamental groups of 4-manifolds and so on.

We will present some applications and relations of L^2-invariants to projective class groups and Grothendieck groups of groups rings. We will explain relations between the Atiyah Conjecture about the possible values of L^2-Betti numbers and the Farrell-Jones Conjecture in algebraic K-theory.